

Capacity and Performance of Adaptive MIMO System Based on Beam-Nulling

Mabruk Gheryani, Zhiyuan Wu, and Yousef R. Shayan
Concordia University, Department of Electrical Engineering
Montreal, Quebec, H4G 2W1, Canada
email: (m_gherya, zy_wu, yshayan)@ece.concordia.ca

Abstract

In this paper, we propose a scheme called “beam-nulling” for MIMO adaptation. In the beam-nulling scheme, the eigenvector of the weakest subchannel is fed back and then signals are sent over a generated subspace orthogonal to the weakest subchannel. Theoretical analysis and numerical results show that the capacity of beam-nulling is closed to the optimal water-filling at medium SNR. Additionally, signal-to-interference-plus-noise ratio (SINR) of MMSE receiver is derived for beam-nulling. Then the paper presents the associated average bit-error rate (BER) of beam-nulling numerically which is verified by simulation. Simulation results are also provided to compare beam-nulling with beamforming. To improve performance further, beam-nulling is concatenated with linear dispersion code. Simulation results are also provided to compare the concatenated beam-nulling scheme with the beamforming scheme at the same data rate. Additionally, the existing beamforming and new proposed beam-nulling can be extended if more than one eigenvector is available at the transmitter. The new extended schemes are called multi-dimensional (MD) beamforming and MD beam-nulling. Theoretical analysis and numerical results in terms of capacity are also provided to evaluate the new extended schemes. Simulation results show that the MD scheme with LDC can outperform the MD scheme with STBC significantly when the data rate is high.

I. INTRODUCTION

Since the discovery of multiple-input-multiple-output (MIMO) capacity [1] [2], a lot of research efforts have been put into this field. It has been recognized that adaptive techniques proposed for single-input-single-output (SISO) channels [3] [4] can also be applied to improve MIMO channel capacity.

The ideal scenario is that the transmitter has full knowledge of channel state information (CSI). Given this perfect CSI feedback, the original MIMO channel can be converted to multiple uncoupled SISO channels via singular value decomposition (SVD) at the transmitter and the receiver [1]. In other words, the original MIMO channel can be decomposed into several orthogonal “spatial subchannels” with various propagation gains.

To achieve better performance, various schemes can be implemented depending on the availability of CSI at the transmitter [5]- [17]. If the transmitter has full knowledge of the channel matrix, i.e., full CSI, the so-called “water-filling” (WF) principle is performed on each spatial subchannel to maximize the channel capacity [1]. This scheme is optimal in this case. Various WF-based schemes have been proposed, such as [9] [11]. For the WF-based scheme, the feedback bandwidth for the full CSI grows with respect to the number of transmit and receive antennas and the performance is often very sensitive to channel estimation errors.

To mitigate these disadvantages, various beamforming (BF) techniques for MIMO channels have also been investigated intensively. In an adaptive beamforming scheme, the complex weights of the transmit antennas are fed back from the receiver. If only one eigenvector can be fed back, eigen-beamforming [12] is optimal. The eigen-beamforming scheme only applies to the strongest spatial subchannel but can achieve full diversity and high signal-to-noise ratio (SNR) [12]. Also, in practice, the eigen-beamforming scheme has to cooperate with the other adaptive parameters to improve performance and/or data rates such as constellation and coding rate. There are also other beamforming schemes based on various criteria. Examples of such schemes are [12] - [22]. Note that the conventional beamforming is optimal in terms

of maximizing the SNR at the receiver. However, it is sub-optimal from the MIMO capacity perspective, since only a single data stream, as opposed to parallel streams, is transmitted through the MIMO channel [13].

In this paper, we propose a new technique called “beam-nulling” (BN). This scheme uses the same feedback bandwidth as beamforming, that is, only one eigenvector is fed back to the transmitter. The beam-nulling transmitter is informed by the weakest spatial subchannel and, where both transmitter and receiver know how to generate the same spatial subspace, sends signals over a generated spatial subspace orthogonal to the weakest subchannel. Although the transmitted symbols are “precoded” according to the feedback, beam-nulling is different from the other existing precoding schemes with limited feedback channel, which are independent of the instantaneous channel but the optimal precoding depends on the instantaneous channel [14] [15].

Using this new techniques instead of the optimal water-filling scheme, the loss of channel capacity can be reduced. This paper also addresses the performance of beam-nulling. To achieve better performance, beam-nulling can be concatenated with the other space-time (ST) coding schemes, such as space-time trellis codes (STTCs) [23], space-time block codes (STBCs) [24] [25] and linear dispersion codes (LDCs) [26]- [29], etc. For simplicity and flexibility, LDCs are preferable. We provide numerical and simulation results are provided to demonstrate the merits of the new proposed scheme. Additionally, if more than one eigenvector, e.g. k eigenvectors, can be available at the transmitter, the existing beamforming scheme and the proposed beam-nulling scheme can be further extended, respectively. The extended schemes will exploit or discard k spatial subchannels and they will be referred to as “multi-dimensional (MD)” beamforming and “multi-dimensional” beam-nulling, respectively.

This paper will be organized as follows. Our channel model is presented in Section II. In Section III, four power allocation strategies, i.e., equal power, water-filling, eigen-beamforming, and a new power allocation strategy called “beam-nulling” are studied and compared in terms of channel capacity. In Section IV, bit error rate (BER) of the proposed beam-nulling scheme using MMSE detector is studied and verified. The proposed scheme is compared with the eigen-beamforming scheme at various data rates in terms of BER. Beam-nulling concatenated with LDC is proposed and evaluated. In Section V, extended adaptive frameworks, i.e., MD beamforming and MD beam-nulling, are proposed. Capacity and performance of these two schemes are discussed and compared. To improve performance further and maintain reasonable complexity, MD schemes concatenated with linear space-time codes, such as STBC and LDC, are proposed and evaluated. Finally, in Section VI, conclusions are drawn.

II. CHANNEL MODEL

In this study, the channel is assumed to be a Rayleigh flat fading channel with N_t transmit and N_r ($N_r \geq N_t$) receive antennas. We denote the complex gain from the transmit antenna n to the receiver antenna m by h_{mn} and collect them to form an $N_r \times N_t$ channel matrix $\mathbf{H} = [h_{mn}]$. The channel is known perfectly at the receiver. The entries in \mathbf{H} are assumed to be independent and identically distributed (*i.i.d.*) symmetrical complex Gaussian random variables with zero mean and unit variance.

The symbol vector at the N_t transmit antennas is denoted by $\mathbf{x} = [x_1, x_2, \dots, x_{N_t}]^T$. According to information theory [30], the optimal distribution of the transmitted symbols is Gaussian. Thus, the elements $\{x_i\}$ of \mathbf{x} are assumed to be *i.i.d.* Gaussian variables with zero mean and unit variance, i.e., $E(x_i) = 0$ and $E|x_i|^2 = 1$.

The singular-value decomposition of \mathbf{H} can be written as

$$\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H \quad (1)$$

where \mathbf{U} is an $N_r \times N_r$ unitary matrix, $\mathbf{\Lambda}$ is an $N_r \times N_t$ matrix with singular values $\{\lambda_i\}$ on the diagonal and zeros off the diagonal, and \mathbf{V} is an $N_t \times N_t$ unitary matrix. For convenience, we assume $\lambda_1 \geq \lambda_2 \dots \geq \lambda_{N_t}$, $\mathbf{U} = [\mathbf{u}_1 \mathbf{u}_2 \dots \mathbf{u}_{N_r}]$ and $\mathbf{V} = [\mathbf{v}_1 \mathbf{v}_2 \dots \mathbf{v}_{N_t}]$. $\{\mathbf{u}_i\}$ and \mathbf{v}_i are column vectors. From equation (1), the original channel can be considered as consisting of uncoupled parallel subchannels. Each

subchannel corresponds to a singular value of \mathbf{H} . In the following context, the subchannel is also referred to as “spatial subchannel”. For instance, one spatial subchannel corresponds to λ_i , \mathbf{u}_i and $\{\mathbf{v}_i\}$.

III. POWER ALLOCATION AMONG SPATIAL SUBCHANNELS

We assume that the total transmitted power is constrained to P . Given the power constraint, different power allocation among spatial subchannels can affect the channel capacity tremendously. Depending on power allocation strategy among spatial subchannels, four schemes are presented which are equal power, water-filling, eigen-beamforming, and the new power allocation which is beam-nulling.

If the transmitter has no knowledge about the channel, the most judicious strategy is to allocate the power to each transmit antenna equally, i.e., equal power. In this case, the received signals can be written as

$$\mathbf{y} = \sqrt{\frac{P}{N_t}} \mathbf{H} \mathbf{x} + \mathbf{z} \quad (2)$$

\mathbf{z} is the additive white Gaussian noise (AWGN) vector with *i.i.d.* symmetrical complex Gaussian elements of zero mean and variance σ_z^2 . The associated ergodic channel capacity can be written as [1]

$$\bar{C}_{eq} = E \left[\sum_{i=1}^{N_t} \log \left(1 + \frac{\rho}{N_t} \lambda_i^2 \right) \right] \quad (3)$$

where $E[\cdot]$ denotes expectation with respect to \mathbf{H} and $\rho = \frac{P}{\sigma_z^2}$ denotes SNR. If the transmitter has full knowledge about the channel, the most judicious strategy is to allocate the power to each spatial subchannel by water-filling principle [1]. In water-filling scheme, the received signals can be written as

$$\tilde{y}_i = \sqrt{P_i} \lambda_i x_i + \tilde{z}_i \quad (4)$$

where $\sum_{i=1}^{N_t} P_i = P$ as a constraint and \tilde{z}_i is the AWGN random variable with zero mean and σ_z^2 variance. Following the method of Lagrange multipliers, P_i can be found [1] and the total ergodic channel capacity is

$$\bar{C}_{wf} = E \left[\sum_{i=1}^{N_t} \log \left(1 + \frac{P_i}{\sigma_z^2} \lambda_i^2 \right) \right] \quad (5)$$

To save feedback bandwidth, beamforming can be considered. For the MIMO model, the optimal beamforming is called “eigen-beamforming” [12], or simply beamforming. We assume one symbol, saying x_1 , is transmitted. At the receiver, the received vector can be written as

$$\mathbf{y}_1 = \sqrt{P} \mathbf{H} \mathbf{v}_1 x_1 + \mathbf{z}_1 \quad (6)$$

where \mathbf{z}_1 is the additive white Gaussian noise vector with *i.i.d.* symmetrical complex Gaussian elements of zero mean and variance σ_z^2 . The associated ergodic channel capacity can be written as

$$\bar{C}_{bf} = E \left[\log \left(1 + \rho \lambda_1^2 \right) \right] \quad (7)$$

The eigen-beamforming scheme can save feedback bandwidth and is optimized in terms of SNR [22]. However, since only one spatial subchannel is considered, this scheme suffers from loss of channel capacity [13], especially when the number of antennas grows.

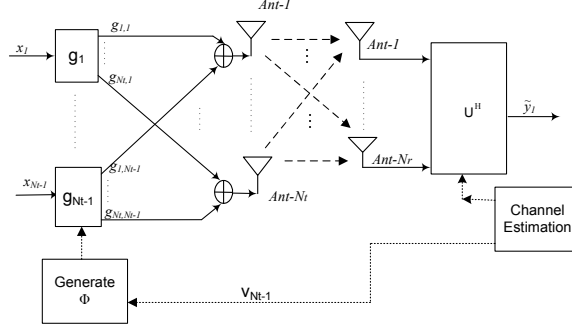


Fig. 1. beam-nulling scheme.

A. Beam-Nulling

The eigen-beamforming scheme can save feedback bandwidth and is optimized in terms of SNR [22]. However, since only a single spatial subchannel is considered, this scheme suffers from loss of channel capacity [13], especially when the number of antennas grows. Inspired by the eigen-beamforming scheme, we will propose a new beamforming-like scheme called “beam-nulling” (BN). This scheme uses the same feedback bandwidth as beamforming, that is, only one eigenvector is fed back to the transmitter. Unlike the eigen-beamforming scheme in which only the best spatial subchannel is considered, the beam-nulling scheme discards only the worst spatial subchannel. Hence, in comparison with the optimal water-filling scheme, the loss of channel capacity can be reduced.

In this scheme as shown in Fig. 1, the eigenvector associated with the minimum singular value from the transmitter side, i.e., \mathbf{v}_{N_t} , is fed back to the transmitter. A subspace orthogonal to the weakest spatial channel is constructed so that the following condition is satisfied.

$$\Phi^H \mathbf{v}_{N_t} = \mathbf{0} \quad (8)$$

The $N_t \times (N_t - 1)$ matrix $\Phi = [\mathbf{g}_1 \mathbf{g}_2 \dots \mathbf{g}_{N_t-1}]$ spans the subspace. Note that the method to construct the subspace Φ should also be known to the receiver.

Here is an example of construction of the orthogonal subspace. We construct an $N_t \times N_t$ matrix

$$\mathbf{A} = [\mathbf{v}_{N_t} \mathbf{I}'] \quad (9)$$

where $\mathbf{I}' = [\mathbf{I}_{(N_t-1) \times (N_t-1)} \mathbf{0}_{(N_t-1) \times 1}]^T$. Applying QR decomposition to \mathbf{A} , we have

$$\mathbf{A} = [\mathbf{v}_{N_t} \Phi] \cdot \Gamma \quad (10)$$

where Γ is an upper triangular matrix with the (1,1)-th entry equal to 1. Φ is the subspace orthogonal to \mathbf{v}_{N_t} .

At the transmitter, $N_t - 1$ symbols denoted as \mathbf{x}' are transmitted over the orthogonal subspace Φ . The received signals at the receiver can be written as

$$\begin{aligned} \mathbf{y}' &= \sqrt{\frac{P}{N_t - 1}} \mathbf{H} \Phi \mathbf{x}' + \mathbf{z}' \\ &= \widehat{\mathbf{H}} \mathbf{x}' + \mathbf{z}' \end{aligned} \quad (11)$$

where \mathbf{z}' is additive white Gaussian noise vector with *i.i.d.* symmetrical complex Gaussian elements of zero mean and variance σ_z^2 and $\widehat{\mathbf{H}} = \sqrt{\frac{P}{N_t - 1}} \mathbf{H} \Phi$.

Substituting (1) into (11) and multiplying \mathbf{y}' by \mathbf{U}^H , results in

$$\tilde{\mathbf{y}} = \sqrt{\frac{P}{N_t - 1}} \Lambda \begin{pmatrix} \mathbf{B} \\ \mathbf{0}^T \end{pmatrix} \mathbf{x}' + \tilde{\mathbf{z}} \quad (12)$$

where $\tilde{\mathbf{z}}$ is additive white Gaussian noise vector with *i.i.d.* symmetrical complex Gaussian elements of zero mean and variance σ_z^2 . With the condition in (8),

$$\mathbf{V}^H \Phi = \begin{pmatrix} \mathbf{B} \\ \mathbf{0}^T \end{pmatrix} \quad (13)$$

where

$$\mathbf{B} = \begin{pmatrix} \mathbf{v}_1^H \mathbf{g}_1 & \mathbf{v}_1^H \mathbf{g}_2 & \cdots & \mathbf{v}_1^H \mathbf{g}_{N_t-1} \\ \mathbf{v}_2^H \mathbf{g}_1 & \ddots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{v}_{N_t-1}^H \mathbf{g}_1 & \cdots & \cdots & \mathbf{v}_{N_t-1}^H \mathbf{g}_{N_t-1} \end{pmatrix} \quad (14)$$

\mathbf{B} is an $(N_t - 1) \times (N_t - 1)$ unitary matrix. From (12), the available spatial channels are $N_t - 1$. Since the weakest spatial subchannel is “nulled” in this scheme, power can be allocated equally among the other $N_t - 1$ subchannels. Equation (12) can be rewritten as

$$\tilde{\mathbf{y}}' = \sqrt{\frac{P}{N_t - 1}} \Lambda' \mathbf{B} \mathbf{x}' + \tilde{\mathbf{z}}' \quad (15)$$

where $\tilde{\mathbf{y}}'$ and $\tilde{\mathbf{z}}'$ are column vectors with the first $(N_t - 1)$ elements of $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{z}}$, respectively, and $\Lambda' = \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_{(N_t-1)}]$. From (15), the associated ergodic channel capacity can be found as

$$\bar{C}_{bn} = E \left[\sum_{i=1}^{N_t-1} \log \left(1 + \frac{\rho}{N_t - 1} \lambda_i^2 \right) \right] \quad (16)$$

As can be seen, the beam-nulling scheme only needs one eigenvector to be fed back. However, since only the worst spatial subchannel is discarded, this scheme can increase channel capacity significantly as compared to the conventional beamforming scheme.

B. Comparisons Among the Four Schemes

In this section, we compare the new proposed beam-nulling scheme with the other schemes, i.e., equal power, beamforming and water-filling schemes. Water-filling is the optimal solution among the four schemes for any SNR.

Differentiating the above ergodic capacities with respect to ρ respectively, we have

$$\frac{\partial \bar{C}_{eq}}{\partial \rho} = E \left[\sum_{i=1}^{N_t} \frac{1}{\rho + \frac{N_t}{\lambda_i^2}} \right] \quad (17)$$

$$\frac{\partial \bar{C}_{bf}}{\partial \rho} = E \left[\frac{1}{\rho + \frac{1}{\lambda_1^2}} \right] \quad (18)$$

$$\frac{\partial \bar{C}_{bn}}{\partial \rho} = E \left[\sum_{i=1}^{N_t-1} \frac{1}{\rho + \frac{N_t-1}{\lambda_i^2}} \right] \quad (19)$$

The differential will also be referred to as “slope”. Since the second order differentials are negative, the above ergodic capacities are concave and monotonically increasing with respect to ρ .

With the fact that $\lambda_1 \geq \lambda_2 \dots \geq \lambda_{N_t}$, it can be readily checked that the slopes of ergodic capacities associate with equal power and beam-nulling are bounded as follows.

$$E \left(\frac{N_t}{\rho + \frac{N_t}{\lambda_1}} \right) \geq \frac{\partial \bar{C}_{eq}}{\partial \rho} \geq E \left(\frac{N_t}{\rho + \frac{N_t}{\lambda_{N_t}}} \right) \quad (20)$$

$$E \left(\frac{N_t - 1}{\rho + \frac{N_t-1}{\lambda_1}} \right) \geq \frac{\partial \bar{C}_{bn}}{\partial \rho} \geq E \left(\frac{N_t - 1}{\rho + \frac{N_t-1}{\lambda_{(N_t-1)}}} \right) \quad (21)$$

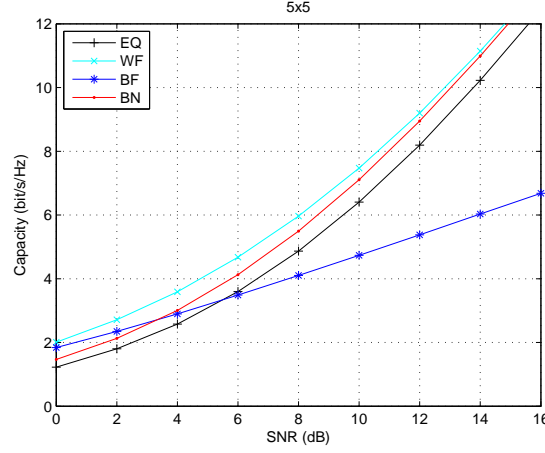


Fig. 2. 5×5 Rayleigh fading channel.

For the case of $N_t = 2$, beamforming and beam-nulling have the same capacity for any ρ as can be seen from equations of capacity and slope. If $\rho \rightarrow 0$, equivalently at low SNR, it can be easily found that

$$\frac{\partial \bar{C}_{bf}}{\partial \rho} \geq \frac{\partial \bar{C}_{bn}}{\partial \rho} \geq \frac{\partial \bar{C}_{eq}}{\partial \rho}, \rho \rightarrow 0 \quad (22)$$

If $\rho \rightarrow \infty$, equivalently at high SNR, it can be easily found that

$$\frac{\partial \bar{C}_{eq}}{\partial \rho} \geq \frac{\partial \bar{C}_{bn}}{\partial \rho} \geq \frac{\partial \bar{C}_{bf}}{\partial \rho}, \rho \rightarrow \infty \quad (23)$$

Note that $\bar{C}_{bf} = \bar{C}_{bn} = \bar{C}_{eq} = 0$ when $\rho = 0$ or minus infinity in dB. Hence, at medium SNR, $\frac{\partial \bar{C}_{bn}}{\partial \rho}$ has the largest value compared to $\frac{\partial \bar{C}_{bf}}{\partial \rho}$ and $\frac{\partial \bar{C}_{eq}}{\partial \rho}$. Therefore, for low, medium and high SNRs, beamforming, beam-nulling and equal power have the largest capacities, respectively.

In Fig. 2, capacities of water-filling, beamforming, beam-nulling and equal power are compared over 5×5 Rayleigh fading channels, respectively. Note that since SNR is measured in dB, the curves become convex. In these figures, “EQ” stands for equal power, “WF” stands for water-filling, “BF” stands for beamforming and “BN” stands for beam-nulling. As can be seen, the water-filling has the best capacity at any SNR region. The other schemes perform differently at different SNR regions. At low SNR, the beamforming is the closest to the optimal water-filling, e.g., the SNR region below 3.5 dB for 5×5 fading channel. Note that at low SNR, the water-filling scheme may only allocate power to one or two spatial subchannels. At medium SNR, the proposed beam-nulling is the closest to the optimal water-filling, e.g., the SNR region from 3.5 dB to 16 dB for 5×5 fading channel. The beam-nulling scheme only discards the weakest spatial subchannel and allocates power to the other spatial subchannels. As can be seen from the numerical results, the beam-nulling scheme performs better than the other schemes in this case. Note that at high SNR, the equal power scheme will converge with the water-filling scheme.

IV. PERFORMANCE OF BEAM-NULLING

A. MMSE Detector

The close-form error probability for the optimal ML receiver is difficult to establish. Other suboptimal receivers can also be implemented. The MMSE detector is especially popular due to its low complexity and good performance [31] [32]. In the following context, BER of the MMSE detector is analyzed for the beam-nulling scheme.

Let us define $\widehat{\mathbf{H}} = \sqrt{\frac{P}{N_t-1}} \mathbf{H} \Phi$ and $\hat{\mathbf{h}}_i$ is the i -th column of $\widehat{\mathbf{H}}$. Equation (11) can also be written as

$$\mathbf{y}' = \hat{\mathbf{h}}_i x_i + \sum_{j \neq i} \hat{\mathbf{h}}_j x_j + \mathbf{z}' \quad (24)$$

where x_i is the i -th element of \mathbf{x}' .

Without loss of generality, we consider the detection of one symbol, say x_i . We collect the rest of the symbols into a column vector \mathbf{x}_I and denote $\widehat{\mathbf{H}}_I = [\hat{\mathbf{h}}_1, \dots, \hat{\mathbf{h}}_{i-1}, \hat{\mathbf{h}}_{i+1}, \dots, \hat{\mathbf{h}}_{N_t-1}]$ as the matrix obtained by removing the i -th column from $\widehat{\mathbf{H}}$.

A linear MMSE detector [32] [33] is applied and the corresponding output is given by

$$\hat{x}_i = \mathbf{w}_i^H \mathbf{y} = x_i + \hat{z}_i. \quad (25)$$

where \hat{z}_i is the noise term of zero mean. \hat{z}_i can be approximated to be Gaussian [32]. The corresponding \mathbf{w}_i can be found as

$$\mathbf{w}_i = \frac{(\hat{\mathbf{h}}_i \hat{\mathbf{h}}_i^H + \mathbf{R}_I)^{-1} \hat{\mathbf{h}}_i}{\hat{\mathbf{h}}_i^H (\hat{\mathbf{h}}_i \hat{\mathbf{h}}_i^H + \mathbf{R}_I)^{-1} \hat{\mathbf{h}}_i} \quad (26)$$

where $\mathbf{R}_I = \widehat{\mathbf{H}}_I \widehat{\mathbf{H}}_I^H + \sigma_z^2 \mathbf{I}$. Note that the scaling factor $\frac{1}{\hat{\mathbf{h}}_i^H (\hat{\mathbf{h}}_i \hat{\mathbf{h}}_i^H + \mathbf{R}_I)^{-1} \hat{\mathbf{h}}_i}$ in the coefficient vector of the MMSE detector \mathbf{w}_i is added to ensure an unbiased detection as indicated by (25). The variance of the noise term \hat{z}_i can be found from (25) and (26) as

$$\hat{\sigma}_i^2 = \mathbf{w}_i^H \mathbf{R}_I \mathbf{w}_i \quad (27)$$

Substituting the coefficient vector for the MMSE detector in (26) into (27), the variance can be written as

$$\hat{\sigma}_i^2 = \frac{1}{\hat{\mathbf{h}}_i^H \mathbf{R}_I^{-1} \hat{\mathbf{h}}_i} \quad (28)$$

Then, the SINR of MMSE associated with x_i is $1/\hat{\sigma}_i^2$.

$$\gamma_i = \frac{1}{\hat{\sigma}_i^2} = \hat{\mathbf{h}}_i^H \mathbf{R}_I^{-1} \hat{\mathbf{h}}_i \quad (29)$$

The closed-form BER for a channel model such as (25) can be found in [34]. The average BER over MIMO fading channel for a given constellation can be found for beam-nulling as follows.

$$BER_{av} = E_{\gamma_i} \left[\frac{1}{N_t - 1} \sum_i BER(\gamma_i) \right] \quad (30)$$

The closed-form formula for the average BER in (30) depends on the distribution of γ_i , which is difficult to determine. Here, the above average BER is calculated numerically. For example, the average BER for 2^η -PSK is

$$BER_{av} = E_{\gamma_i} \left[\frac{1}{N_t - 1} \sum_i \frac{2}{\eta} Q \left(\sqrt{2\eta \gamma_i} \sin\left(\frac{\pi}{2^\eta}\right) \right) \right] \quad (31)$$

and the average BER for rectangular 2^η -QAM is

$$BER_{av} = E_{\gamma_i} \left[\frac{1}{N_t - 1} \sum_i \frac{4}{\eta} Q \left(\sqrt{\frac{3\eta \gamma_i}{2^\eta - 1}} \right) \right] \quad (32)$$

where $Q(\cdot)$ denotes the Gaussian Q -function.

In Fig. 3, numerical and simulation results are compared for 8PSK over 3×3 Rayleigh fading channel and QPSK over 4×4 Rayleigh fading channel, respectively. As can be seen, the numerical and simulation results match well.

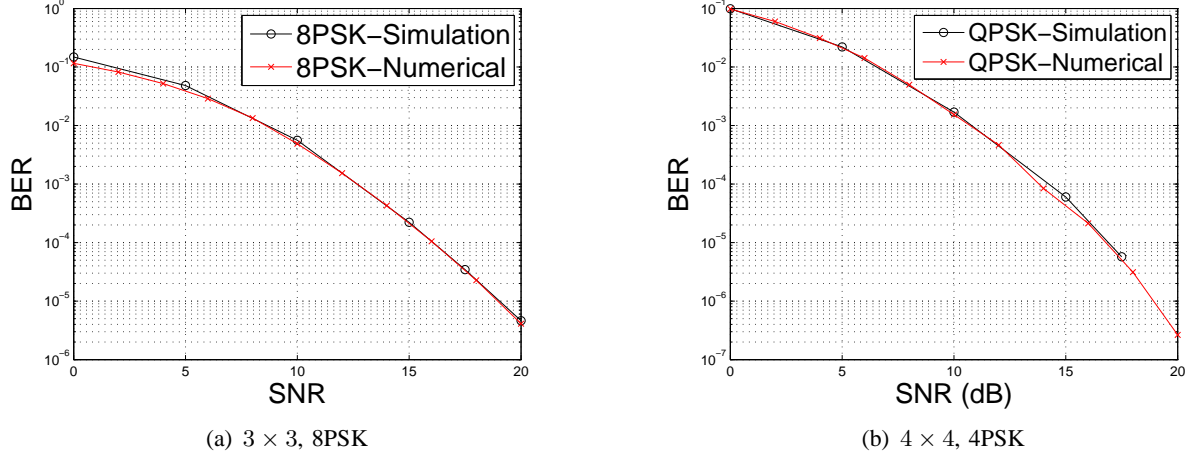


Fig. 3. Numerical and simulation results for beam-nulling scheme.

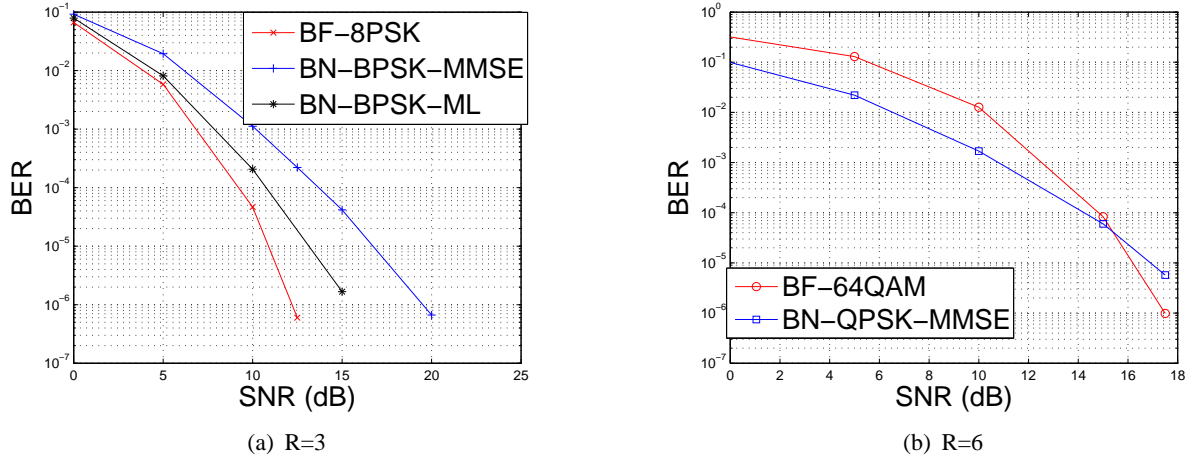


Fig. 4. Comparison over 4×4 Rayleigh fading channel.

B. Performance Comparison Between Beamforming and Beam-nulling

In Fig. 4, simulation results are compared for various data rates R over 4×4 Rayleigh fading channels. In the following simulations, a data rate R is measured in bits per channel use. The beamforming scheme is equivalent to a SISO channel using a maximum ratio combining (MRC) receiver [14]. For the beam-nulling scheme, the optimal ML receiver and the suboptimal MMSE receiver are used.

From Fig. 4, if the data rate is low, i.e., constellation size is low, beamforming outperforms beam-nulling. If the data rate is high, i.e., constellation size is high, beam-nulling outperforms beamforming at low and medium SNR, however at high SNR beamforming outperforms beam-nulling. Also, as can be seen, at the high data rate, even the beam-nulling scheme with suboptimal MMSE receiver outperforms the beamforming scheme.

C. Concatenation of Beam-nulling and LDC

To further improve the performance of beam-nulling with tractable complexity, we propose to concatenate beam-nulling with a linear dispersion code. Note that to meet error-rate requirements, multiple levels of error protection can be implemented. In this study, we focus on space-time coding domain.

In this system, the information bits are first mapped into symbols. The symbol stream is parsed into blocks of length $L = (N_t - 1)T$. The symbol vector associated with one modulation block is denoted by

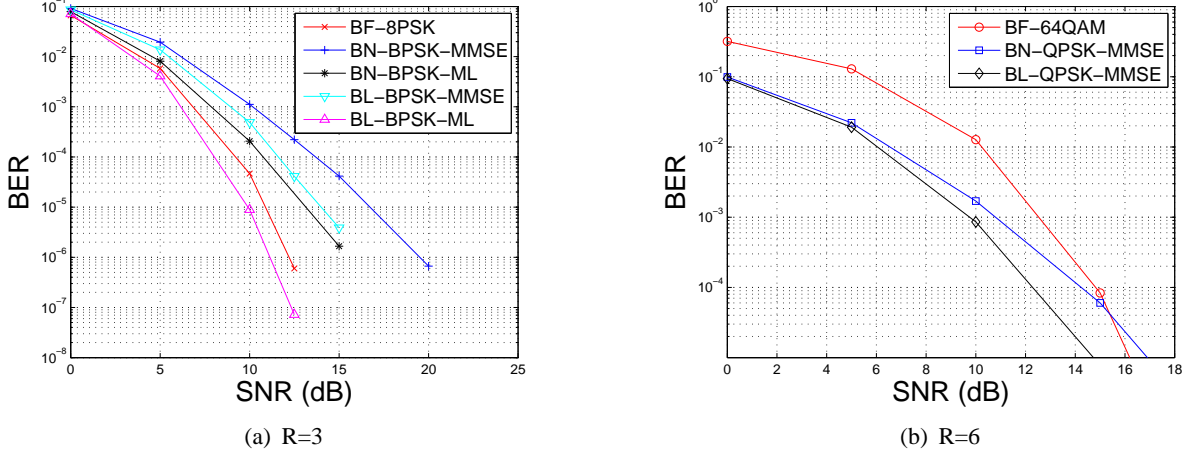


Fig. 5. Comparison over 4×4 Rayleigh fading channel.

$\mathbf{x} = [x_1, x_2, \dots, x_L]^T$ with $x_i \in \Omega \equiv \{\Omega_m | m = 0, 1, \dots, 2^\eta - 1, \eta \geq 1\}$, i.e., a complex constellation of size 2^η , such as 2^η -QAM). The average symbol energy is assumed to be 1, i.e., $\frac{1}{2^\eta} \sum_{m=0}^{2^\eta-1} |\Omega_m|^2 = 1$. Each symbol in a block will be mapped to a dispersion matrix of size $N_t \times T$ (i.e., \mathbf{M}_i) and then combined linearly to form $(N_t - 1)$ data streams over T channel uses. The output $(N_t - 1)$ data streams are transmitted only over the subspace Φ orthogonal to the weakest spatial channel. The generation of the orthogonal subspace Φ is described in Section III-A. The received signals can be written as

$$\mathbf{y} = \sqrt{\frac{P}{N_t - 1}} \mathbf{H} \Phi \sum_{i=1}^L \mathbf{M}_i x_i + \mathbf{z} \quad (33)$$

where \mathbf{z} is additive white Gaussian noise vector with *i.i.d.* symmetrical complex Gaussian elements of zero mean and variance σ_z^2 . It is worthy to note that the traditional beamforming scheme cannot work with space-time coding since it can be viewed as a SISO channel. We compare the concatenated scheme with the original schemes at the same data rate.

In Fig. 5, simulation results are compared for various data rates R over 4×4 Rayleigh flat fading channels. In the figure, “BL” denotes beam-nulling with LDC. As can be seen, beam-nulling with LDC outperforms beam-nulling without LDC using the same receiver. The performance of beam-nulling with LDC using MMSE receiver is close to that of beam-nulling without LDC using the optimal ML receiver.

Also it can be seen, if data rate is low, i.e., constellation size is low, the performance of beam-nulling with LDC can approach that of beamforming at high SNR. If data rate is high, i.e., constellation size is high, beam-nulling with LDC outperforms beamforming even when the suboptimal MMSE receiver is used.

V. EXTENDED ADAPTIVE FRAMEWORKS

For the beamforming and beam-nulling schemes, only one eigenvector has been fed back to the transmitter. If more backward bandwidth is available for feedback, e.g. k eigenvectors, can be sent to the transmitter for adaptation. With the feedback of k eigenvectors, we can extend our frameworks, which will be called multi-dimensional (MD) beamforming and MD beam-nulling. The original schemes can be referred to as 1D-beamforming and 1D-beam-nulling. To save bandwidth, $k \leq \lfloor \frac{N_t}{2} \rfloor$ should be satisfied, where $\lfloor \cdot \rfloor$ denotes rounding towards minus infinity. That is, whether the strongest or the weakest k spatial subchannels will be fed back according to the channel conditions. For example, at low SNR, k strongest spatial subchannels will be fed back. At medium SNR, k weakest spatial subchannels will be fed back.

A. MD Beamforming

For MD beamforming, $\mathbf{v}_1, \dots, \mathbf{v}_k$ are fed back to the transmitter. k symbols, saying $\mathbf{x}_k = [x_1, x_2, \dots, x_k]^T$, are transmitted. At the receiver, the received vector can be written as

$$\mathbf{y}_k = \sqrt{\frac{P}{k}} \mathbf{H}[\mathbf{v}_1 \dots \mathbf{v}_k] \mathbf{x}_k + \mathbf{z}_k \quad (34)$$

where \mathbf{z}_k is the additive white Gaussian noise vector with *i.i.d.* symmetrical complex Gaussian elements of zero mean and variance σ_z^2 .

Consequently, the associated ergodic channel capacity can be found as

$$\bar{C}_{k,bf} = E \left[\sum_{i=1}^k \log \left(1 + \frac{P}{k \sigma_z^2} \lambda_i^2 \right) \right] \quad (35)$$

Let $\rho = P/\sigma_z^2$ denote SNR. It is readily checked that the capacity of MD beamforming is also concave and monotonically increasing with respect to SNR ρ . Differentiating the above ergodic capacity with respect to ρ , we have

$$\frac{\partial \bar{C}_{k,bf}}{\partial \rho} = E \left(\sum_{i=1}^k \frac{1}{\rho + \frac{k}{\lambda_i^2}} \right) \quad (36)$$

If $\rho \rightarrow 0$, equivalently at low SNR, it can be easily found that

$$\frac{\partial \bar{C}_{(k-1),bf}}{\partial \rho} > \frac{\partial \bar{C}_{k,bf}}{\partial \rho}, \rho \rightarrow 0 \quad (37)$$

If $\rho \rightarrow \infty$, equivalently at high SNR, it can be easily found that

$$\frac{\partial \bar{C}_{k,bf}}{\partial \rho} > \frac{\partial \bar{C}_{(k-1),bf}}{\partial \rho}, \rho \rightarrow \infty \quad (38)$$

Note that $\bar{C}_{k,bf} = 0$ for any k when $\rho = 0$ or minus infinity in dB. Hence, at low SNR, the capacity of the k -D beamforming scheme is worse than the $(k-1)$ -D beamforming scheme and while at high SNR, the capacity of the k -D beamforming scheme is better than the $(k-1)$ D beamforming scheme at the cost of feedback bandwidth.

B. MD Beam-nulling

For MD beam-nulling, similar to 1D beam-nulling, by a certain rule, a subspace orthogonal to the k weakest spatial channel is constructed. That is, the following condition should be satisfied.

$$\mathbf{v}_n^H \Phi^{(k)} = \mathbf{0}^T, \forall n = N_t - k + 1, \dots, N_t. \quad (39)$$

The $N_t \times (N_t - k)$ matrix $\Phi^{(k)} = [\mathbf{g}_1 \mathbf{g}_2 \dots \mathbf{g}_{N_t-k}]$ spans the $(N_t - k)$ -dimensional subspace.

At the transmitter, $N_t - k$ symbols denoted as $\mathbf{x}^{(k)}$ are transmitted only over the orthogonal subspace $\Phi^{(k)}$. The received signals at the receiver can be written as

$$\mathbf{y}^{(k)} = \sqrt{\frac{P}{N_t - k}} \mathbf{H} \Phi^{(k)} \mathbf{x}^{(k)} + \mathbf{z}^{(k)} \quad (40)$$

where $\mathbf{z}^{(k)}$ is additive white Gaussian noise vector with *i.i.d.* symmetrical complex Gaussian elements of zero mean and variance σ_z^2 . From (40), the associated instantaneous channel capacity with respect to \mathbf{H} can be found as

$$\bar{C}_{bn}^{(k)} = E \left[\sum_{i=1}^{N_t-k} \log \left(1 + \frac{P}{(N_t - k) \sigma_z^2} \lambda_i^2 \right) \right] \quad (41)$$

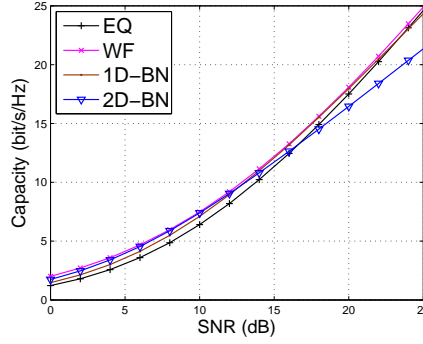


Fig. 6. MD beam-nulling over 5×5 Rayleigh fading channel.

It is readily checked that the capacity of MD beam-nulling is also concave and monotonically increasing with respect to SNR ρ . Let $\rho = P/\sigma_z^2$ denote SNR. Differentiating the above ergodic capacity with respect to ρ , we have

$$\frac{\partial \bar{C}_{bn}^{(k)}}{\partial \rho} = E \left(\sum_{i=1}^{N_t-k} \frac{1}{\rho + \frac{N_t-k}{\lambda_i^2}} \right) \quad (42)$$

If $\rho \rightarrow 0$, equivalently at low SNR, it can be easily found that

$$\frac{\partial \bar{C}_{bn}^{(k)}}{\partial \rho} > \frac{\partial \bar{C}_{bn}^{(k-1)}}{\partial \rho}, \rho \rightarrow 0 \quad (43)$$

If $\rho \rightarrow \infty$, equivalently at high SNR, it can be easily found that

$$\frac{\partial \bar{C}_{bn}^{(k-1)}}{\partial \rho} > \frac{\partial \bar{C}_{bn}^{(k)}}{\partial \rho}, \rho \rightarrow \infty \quad (44)$$

Note that $\bar{C}_{k,bn} = 0$ for any k when $\rho = 0$ or minus infinity in dB. Hence, at low SNR, the capacity of the k -D beam-nulling scheme is better than the $(k-1)$ -D beam-nulling scheme at the cost of feedback bandwidth and while at high SNR, the capacity of the k -D beam-nulling scheme is worse than the $(k-1)$ -D beam-nulling scheme.

For example, in Fig. 6, capacities of 1D beam-nulling and 2D beam-nulling schemes are compared with WF and equal power scheme over 5×5 Rayleigh fading channel at different SNR regions. At relatively low SNR, i.e., less than 13dB, the 2D beam-nulling scheme outperforms the 1D beam-nulling scheme in terms of capacity at the price of feedback bandwidth. While at relatively high SNR, i.e., more than 13dB, the 1D-beam-nulling scheme outperforms the 2D beam-nulling scheme as predicted.

C. Capacity Comparison of MD Schemes

Here, over 5×5 Rayleigh fading channel, the MD schemes are compared with WF and equal power schemes as shown in Fig. 7. It can be readily check that, at relatively low SNR, MD beamforming schemes are better than MD beam-nulling schemes; while at relatively high SNR, the results are opposite. Specifically, at very low SNR, i.e. less than 0dB, the 1D beamforming scheme outperforms the other MD schemes. At the SNR region between 0dB and 5.5dB, the 2D beamforming scheme outperforms the other MD schemes. At the SNR region between 5.5dB and 12.7dB, the 2D beam-nulling scheme outperforms the other MD schemes. At the SNR region between 12.7dB and 23dB, the 1D beam-nulling scheme outperforms the other MD schemes. Again, when SNR is more than 23dB, the equal power scheme outperforms the other suboptimal schemes.

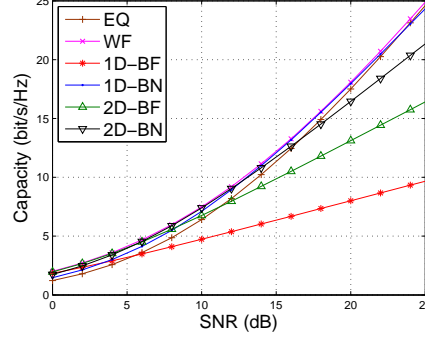


Fig. 7. Comparison over 5×5 Rayleigh fading channel.

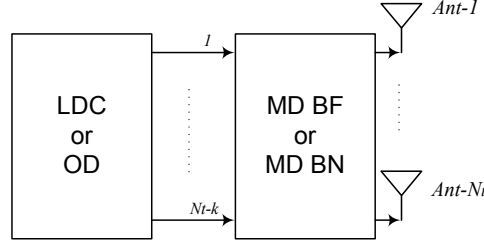


Fig. 8. Concatenated MD scheme.

D. MD Schemes Concatenated with Linear Space-Time Code

MD beamforming scheme and MD beam-nulling scheme make k and $N_t - k$ spatial subchannels available, respectively. As a result, they can concatenate with space-time schemes to improve performance. For simplicity, space-time codes with linear structure, such as high-rate LDCs [26] and STBCs [25] (i.e., orthogonal design), are preferable. It is worthy of noting that the 2D beamforming scheme in [12] is just a special case of MD beamforming. As shown in Fig. 8, we propose to concatenate an MD scheme with an LDC or an STBC. In these figures “OD” stands for orthogonal design.

Over 5×5 Rayleigh fading channel, concatenated MD schemes are compared at various data rate. In the simulations, two eigenvectors can be fed back to the transmitter. For an MD scheme with LDC, a suboptimal linear MMSE receiver is applied. Since a MD scheme with STBC are orthogonal, a matched filter is applied, which is also optimal.

In Fig. 9, MD beamforming scheme with STBC are compared with MD beamforming scheme with LDC in terms of BER when data rate is $R = 2$. Also when $R = 6$, Their BERs are shown in Fig. 11. From these figures, it is shown that at high data rate, MD beamforming with LDC outperform MD beamforming with STBC significantly even though a suboptimal MMSE receiver is applied. Specifically, when BER is 10^{-5} , the coding gain is about 4dB. At low data rate, MD beamforming with LDC performs slightly worse than MD beamforming with STBC since the suboptimal receiver is applied. Specifically, when BER is 10^{-5} , the coding gain is about 1dB.

In Fig. 10, MD beamforming scheme with STBC are compared with MD beamforming scheme with LDC in terms of BER when data rate is $R = 3$. Also when $R = 6$, Their BERs are shown in Fig. 11. From these figures, it is shown that at high data rate, MD beam-nulling with LDC outperform MD beam-nulling with STBC significantly even though a suboptimal MMSE receiver is applied. Specifically, when BER is 10^{-5} , the coding gain is about 6.8dB. At low data rate, MD beam-nulling with LDC performs slightly worse than MD beam-nulling with STBC since the suboptimal receiver is applied. Specifically, when BER is 10^{-5} , the coding gain is about 1.5dB.

In Fig. 11, four schemes are compared when data rate is $R = 6$. As shown in the figure, MD beam-nulling with LDC has the best BER performance even suboptimal MMSE receiver is used. In summary,

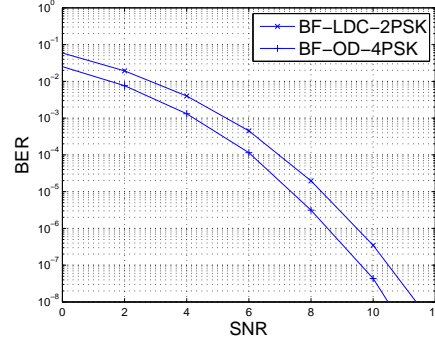


Fig. 9. BER of concatenated MD beamforming when $R = 2$.

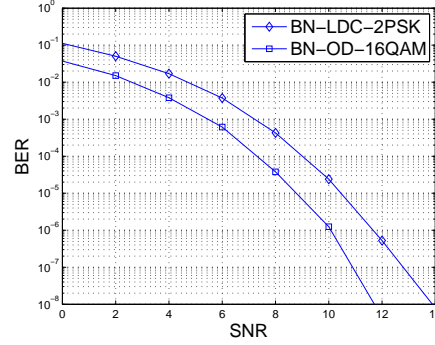


Fig. 10. BER of concatenated MD beam-nulling when $R = 3$.

MD scheme with LDC outperforms MD scheme with STBC especially when the data rate is high. At low data rate, the performance will depend on the receiver. At high data rate, MD beam-nulling with LDC perform the best among the four schemes.

VI. CONCLUSIONS

Based on the concept of spatial subchannels and inspired by the beamforming scheme, we proposed a scheme called “beam-nulling”. The new scheme exploits all spatial subchannels except the weakest one and thus achieves significantly high capacity that approaches the optimal water-filling scheme at medium signal-to-noise ratio. The performance of beam-nulling with an MMSE receiver has been analyzed and verified by numerical and simulation results. It has been shown that if the data rate is low, beamforming outperforms beam-nulling. If the data rate is high, beam-nulling outperforms beamforming at low and

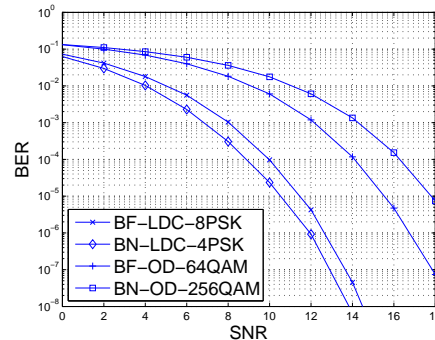


Fig. 11. BER Comparison of concatenated MD schemes when $R = 6$.

medium SNR but beamforming outperforms at high SNR. To achieve better performance and maintain tractable complexity, beam-nulling was concatenated with a linear dispersion code and it was demonstrated that if the data rate is low, beam-nulling with a linear dispersion code can approach beamforming at high SNR. If the data rate is high, beam-nulling outperforms beamforming even with a suboptimal MMSE receiver. If more than one eigenvector can be fed back to the transmitter, new extended schemes based on the existing beamforming and the proposed beam-nulling are proposed. The new schemes are called multi-dimensional beamforming and multi-dimensional beam-nulling, respectively. The theoretical analysis and numeric results in terms of capacity are also provided to evaluate the new proposed schemes. Both of MD schemes can be concatenated with an LDC or an STBC. It is shown that the MD scheme with LDC can outperform the MD scheme with STBC significantly when the data rate is high. Additionally, at high data rate, MD beam-nulling with LDC outperforms MD beamforming with LDC, MD beamforming with STBC and MD beam-nulling with STBC.

REFERENCES

- [1] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecom.*, vol 10, pp. 585-595, Nov. 1999.
- [2] G. J. Foschini, M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, no. 3, pp. 311-335, 1998.
- [3] J. K. Cavers, "Variable-rate transmission for Rayleigh fading channels," *IEEE Transactions on Communications*, COM-20, pp.15-22, 1972.
- [4] A. J. Goldsmith and S.-G. Chua, "Variable rate variable power MQAM for fading channels," *IEEE Trans. Commun.*, vol. 45, no. 10, pp. 1218-1230, Oct. 1997.
- [5] Z. Luo, H. Gao, Y. Liu and J. Gao "Capacity Limits of Time-Varying MIMO Channels," *IEEE International Conference On Communications* vol.2, pp. 795- 799, May 2005.
- [6] W. Yu, W. Rhee, S. Boyd, and J. Cioffi, "Iterative water-filling for Gaussian vector multiple-access channels," *IEEE Trans. Inform. Theory*, vol.50, no. 1, pp. 145-152, Jan. 2004.
- [7] N. Jindal, W. Rhee, S. Vishwanath, S. A. Jafar, and A. Goldsmith, "Sum power iterative water-filling for multi-antenna gaussian broadcast channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 4, pp. 1570-1580, April 2005.
- [8] M. Demirkol and M. Ingram, "Power-controlled capacity for interfering MIMO links," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Atlantic City, USA, Oct. 2001, pp. 187-191.
- [9] Z. Shen, R. W. Heath, Jr., J. G. Andrews, and B. L. Evans, "Comparison of Space-Time Water-filling and Spatial Water-filling for MIMO Fading Channels," in *Proc. IEEE Int Global Communications Conf.* vol. 1, pp. 431 - 435, Nov. 29-Dec. 3, 2004, Dallas, TX, USA.
- [10] Z. Zhou and B. Vucetic "Design of adaptive modulation using imperfect CSI in MIMO systems," *2004 Electronics Letters* vol. 40 no. 17, Aug. 2004.
- [11] X. Zhang and B. Ottersten, "Power allocation and bit loading for spatial multiplexing in MIMO systems," *IEEE Int. Conf.on Acoustics, Speech, and Signal Processing, 2003. Proceedings (ICASSP '03)* vol.5 pp. 54-56, Apr. 2003.
- [12] S. Zhou and G. B. Giannakis, "Optimal transmitter eigen-beamforming and space-time block coding based on channel mean feedback" *IEEE Transactions on Signal Processing*, vol. 50, no. 10, October 2002.
- [13] S. Zhou and G. B. Giannakis, "How accurate channel prediction needs to be for transmit-beamforming with adaptive modulation over Rayleigh MIMO channels," *IEEE Trans. Wireless Comm.*, vol. 3, no. 4, pp. 1285 - 1294, July 2004.
- [14] D. J. Love, R. W. Heath, Jr. and T. Strohmer, "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2735 - 2747, Oct. 2003.
- [15] J. Zheng and B. D. Rao, "Capacity analysis of MIMO systems using limited feedback transmit precoding schemes," *IEEE Trans. on Signal Processing*, vol. 56, no. 7, pp. 2886 - 2901, July 2008.
- [16] S. Zhou and G. B. Giannakis, "Adaptive modulation for multiantenna transmissions with channel mean feedback," *IEEE Trans. Wireless Comm.*, vol.3, no.5, pp. 1626-1636, Sep. 2004.
- [17] P. Xia and G. B. Giannakis, "Multiantenna adaptive modulation with beamforming based on bandwidth-constrained feedback," *IEEE Transactions on Communications*, vol.53, no.3, March 2005.
- [18] B. Mondal and R. W. Heath, Jr., "Performance analysis of quantized beamforming MIMO systems," *IEEE Transactions on Signal Processing*, vol. 54, no. 12, Dec. 2006.
- [19] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE J. Select. Areas in Commun.*, vol. 24, no. 3, pp. 528-541, March 2006.
- [20] S. Zhou, Z. Wang, and G. Giannakis, "Quantifying the power loss when transmit beamforming relies on finite rate feedback," *IEEE Trans. on Wireless Commun.*, vol. 4, no. 4, pp. 1948-1957, 2005.
- [21] J. F. Paris and A. J. Goldsmith, "Adaptive Modulation for MIMO Beamforming under Average BER Constraints and Imperfect CSI," *Proc. of Int. Conf. Comm. ICC 2006*, pp.1312-1317, June 2006.
- [22] J. K. Cavers, "Single-user and multiuser adaptive maximal ratio transmission for Rayleigh channels," *IEEE Trans. Veh. Technol.*, vol. 49, no. 6, pp. 2043-2050, Nov. 2000.
- [23] V. Tarokh, N. Seshadri, and A. Calderbank, "Space-time codes for high data rate wireless communications: Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744-765, Mar. 1998.
- [24] S. Alamouti, "A simple transmitter diversity scheme for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451-1458, Oct. 1998.

- [25] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block code from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456-1467, July 1999.
- [26] B. Hassibi and B. Hochwald, "High-rate codes that are linear in space and time," *IEEE Trans. Inform. Theory*, vol. 48, pp. 1804-1824, July 2002.
- [27] R. W. Heath and A. Paulraj, "Linear dispersion codes for MIMO systems based on frame theory," *IEEE Trans. on Signal Processing*, vol. 50, No. 10, pp. 2429-2441, October 2002.
- [28] X. Ma and G. B. Giannakis, "Full-diversity full-rate complex-field space-time coding," *IEEE Trans. Signal Processing*, vol. 51, no. 11, pp. 2917-2930, July 2003.
- [29] Z. Wu and X. F. Wang, "Design of coded space-time modulation," *IEEE International Conference on Wireless Networks, Communications and Mobile Computing*, vol. 2, pp. 1059-1064, Jun. 13-16, 2005.
- [30] C. E. Shannon, "A mathematical theory of communication, *Bell Syst. Tech. J.*, vol. 27, pp. 379-423 (Part one), pp. 623-656 (Part two), Oct. 1948, reprinted in book form, University of Illinois Press, Urbana, 1949.
- [31] R. Lupas and S. Verdu, "Linear multiuser detectors for synchronous code-division multiple-access channels," *IEEE Trans. inform. Theory*, vol. 35, pp. 123-136, Jan. 1989.
- [32] H. V. Poor and S. Verdu, "Probability of error in MMSE multiuser detection," *IEEE Trans. inform. Theory*, vol. 43, pp. 858-871, May 1997.
- [33] R. Bohnke and K. Kammeyer, "SINR Analysis for V-BLAST with Ordered MMSE-SIC Detection," *International Wireless Communications and Mobile Computing Conference*, pp. 623-628, July 2006.
- [34] J. Proakis, *Digital Communications*, 4th ed. New York: McGraw-Hill, 2001.